

# Supplement 5

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Decision Theory

# Decision Theory

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Decision theory represents a general approach to decision making.

- A set of possible future conditions.
- A list of alternatives for the manager to choose from.
- A known payoff for each alternative under each possible future condition.

# Decision Theory

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A decision maker-

- Identify the possible future conditions.
- Develop a list of possible alternatives.
- Determine or estimate the payoff associated with each alternative for every possible future condition.
- Estimate the likelihood of each possible future condition.
- Evaluate alternatives according to some decision criterion.

# Payoff table

Table showing the expected payoffs for each alternative in every possible state of the nature.

<b>Alternatives</b>	<b>POSSIBLE FUTURE DEMAND</b>		
	<b>Low</b>	<b>Moderate</b>	<b>High</b>
Small facility	\$10*	\$10	\$10
Medium facility	7	12	12
Large facility	(4)	2	16

\*Present value in \$ millions.

# Steps in the Decision Process

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1. Identify the problem
2. Specify objectives and criteria for a solution
3. Develop suitable alternatives
4. Analyze and compare alternatives
5. Select the best alternative
6. Implement the solution
7. Monitor to see that the desired result is achieved

# Causes of Poor Decisions

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- Decisions occasionally turn out poorly due to unforeseeable circumstances; however, this is not the norm.
- More frequently poor decisions are the result of a combination of
  - Mistakes in the decision process
  - Bounded rationality
  - Suboptimization

# Mistakes in the Decision Process

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- Errors in the Decision Process
  - Failure to recognize the importance of each step
  - Skipping a step
  - Failure to complete a step before jumping to the next step
  - Failure to admit mistakes
  - Inability to make a decision

# Decision Environments

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- There are three general environment categories:
  - **Certainty**
    - Environment in which relevant parameters have known values
  - **Risk**
    - Environment in which certain future events have probabilistic outcomes
  - **Uncertainty**
    - Environment in which it is impossible to assess the likelihood of various possible future events

# Decision Making Under Certainty

Determine the best alternative in the payoff table on the previous page for each of the cases: It is known with certainty that demand will be ( *a* ) low, ( *b* ) moderate, ( *c* ) high.

POSSIBLE FUTURE DEMAND			
Alternatives	Low	Moderate	High
Small facility	\$10*	\$10	\$10
Medium facility	7	12	12
Large facility	(4)	2	16

\*Present value in \$ millions.

# Decision Making Under Certainty

Determine the best alternative in the payoff table on the previous page for each of the cases: It is known with certainty that demand will be ( *a* ) low, ( *b* ) moderate, ( *c* ) high.

- Choose the alternative with the highest payoff. Thus, if we know demand will be low, we would elect to build the small facility and realize a payoff of \$10 million.
- If we know demand will be moderate, a medium factory would yield the highest payoff (\$12 million versus either \$10 million or \$2 million).
- For high demand, a large facility would provide the highest payoff.

# Decision Making Under Uncertainty

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- Decisions are sometimes made under complete uncertainty: No information is available on how likely the various states of nature are.
- Decision Criteria:
  - **Maximin**
    - Choose the alternative with the best of the worst possible payoffs
  - **Maximax**
    - Choose the alternative with the best possible payoff
  - **Laplace**
    - Choose the alternative with the best average payoff
  - **Minimax regret**
    - Choose the alternative that has the least of the worst regrets

# Decision Making Under Uncertainty

Referring to the previous payoff table, determine which alternative would be chosen under each of these strategies:

a. Maximin

b. Maximax

c. Laplace

POSSIBLE FUTURE DEMAND			
Alternatives	Low	Moderate	High
Small facility	\$10*	\$10	\$10
Medium facility	7	12	12
Large facility	(4)	2	16

\*Present value in \$ millions.

a. Using maximin, the worst payoffs for the alternatives are as follows:

Small facility: \$10 million

Medium facility: 7 million

Large facility: 4 million

Hence, since \$10 million is the best, choose to build the small facility using the maximin strategy.

# Decision Making Under Uncertainty

Referring to the previous payoff table, determine which alternative would be chosen under each of these strategies:

a. Maximin

b. Maximax

c. Laplace

Alternatives	POSSIBLE FUTURE DEMAND		
	Low	Moderate	High
Small facility	\$10*	\$10	\$10
Medium facility	7	12	12
Large facility	(4)	2	16

\*Present value in \$ millions.

b. Using maximax, the best payoffs are as follows:

Small facility: \$10 million

Medium facility: 12 million

Large facility: 16 million

The best overall payoff is the \$16 million in the third row. Hence, the maximax criterion leads to building a large facility.

# Decision Making Under Uncertainty

Referring to the previous payoff table, determine which alternative would be chosen under each of these strategies:

a. Maximin

b. Maximax

c. Laplace

c. For the Laplace criterion, first find the row totals, and then divide each of those amounts by the number of states of nature (three in this case). Thus, we have

POSSIBLE FUTURE DEMAND			
Alternatives	Low	Moderate	High
Small facility	\$10*	\$10	\$10
Medium facility	7	12	12
Large facility	(4)	2	16

\*Present value in \$ millions.

	Row Total (in \$ millions)	Row Average (in \$ millions)
Small facility	\$30	\$10.00
Medium facility	31	10.33
Large facility	14	4.67

Because the medium facility has the highest average, it would be chosen under the Laplace criterion.

# Decision Making Under Uncertainty

Referring to the previous payoff table, determine which alternative would be chosen under minimax regret strategy.

- The first step in this approach is to prepare a table of **regrets** (or **opportunity losses** ) . To do this, subtract every payoff *in each column* from the best payoff in that column.

Alternatives	REGRETS (IN \$ MILLIONS)			
	Low	Moderate	High	Worst
Small facility	\$0	\$2	\$6	\$6
Medium facility	3	0	4	4
Large facility	14	10	0	14

# Decision Making Under Uncertainty

Referring to the previous payoff table, determine which alternative would be chosen under minimax regret strategy.

Alternatives	REGRETS (IN \$ MILLIONS)			
	Low	Moderate	High	Worst
Small facility	\$0	\$2	\$6	\$6
Medium facility	3	0	4	4
Large facility	14	10	0	14

- The second step is to identify the worst regret for each alternative. For the first alternative, the worst is 6; for the second, the worst is 4; and for the third, the worst is 14.
- The best of these worst regrets would be chosen using minimax regret. The lowest regret is 4, which is for a medium facility. Hence, that alternative would be chosen.

# Decision Making Under Risk

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- Decisions made under the condition that the probability of occurrence for each state of nature can be estimated
- A widely applied criterion is expected monetary value (EMV)
  - **EMV**
    - Determine the expected payoff of each alternative, and choose the alternative that has the best expected payoff
  - This approach is most appropriate when the decision maker is neither risk averse nor risk seeking

# Example – EMV

Alternatives	Possible Future Demand		
	Low (.30)	Moderate (.50)	High (.20)
Small Facility	\$10	\$10	\$10
Medium Facility	7	12	12
Large Facility	(4)	2	16

$$EMV_{\text{small}} = .30(10) + .50(10) + .20(10) = 10$$

$$EMV_{\text{medium}} = .30(7) + .50(12) + .20(12) = 10.5$$

$$EMV_{\text{large}} = .30(-4) + .50(2) + .20(16) = \$3$$

Build a medium facility

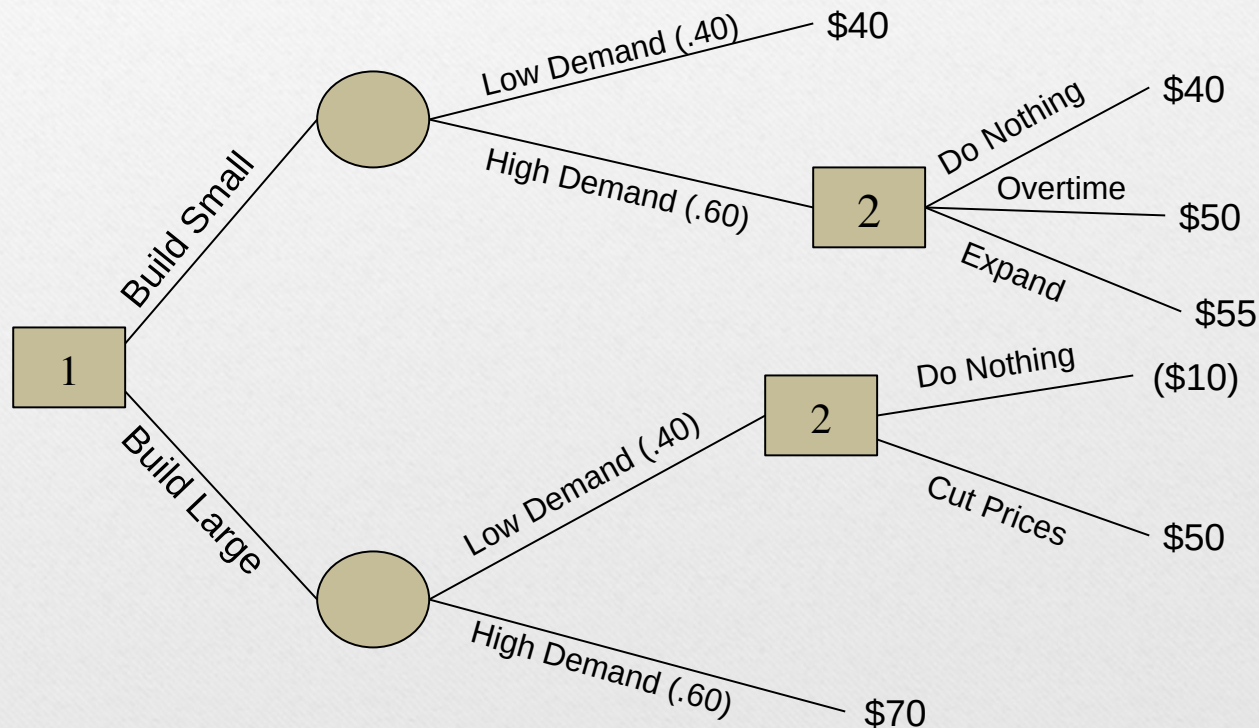
# Decision Tree

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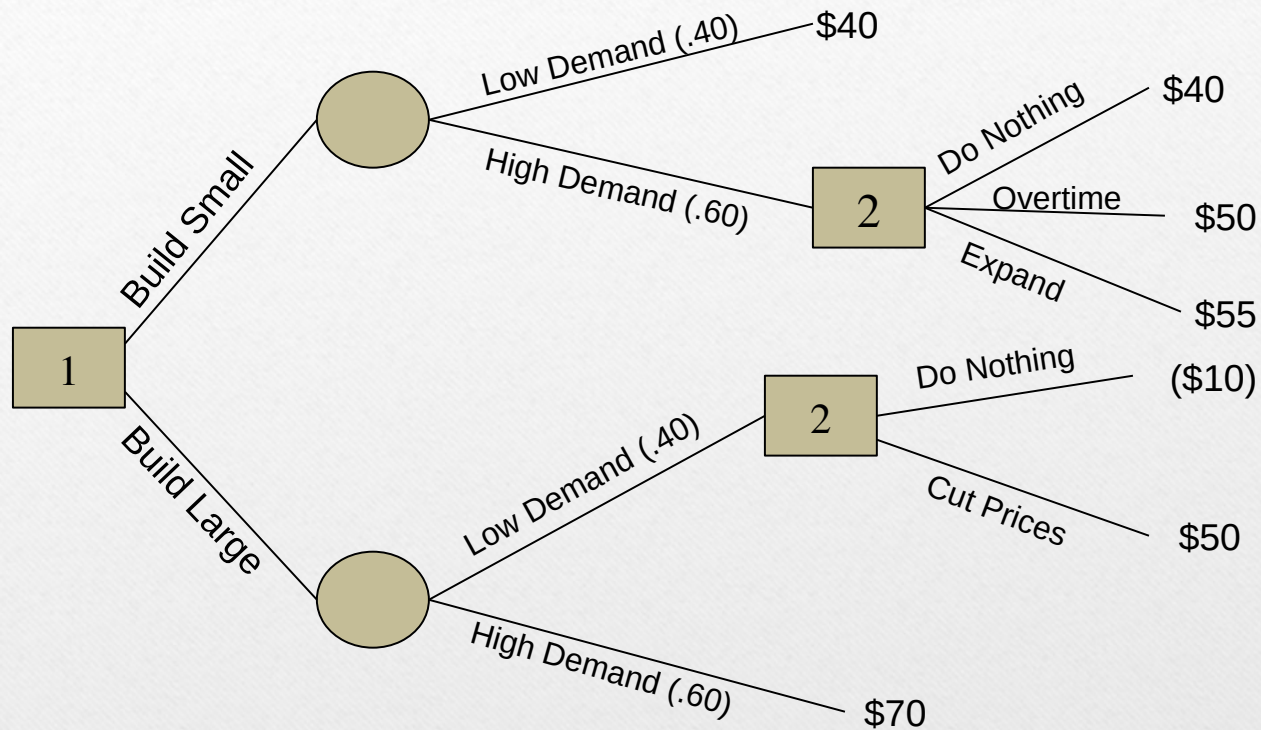
- Composed of
  - Nodes
    - Decisions – represented by square nodes
    - Chance events – represented by circular nodes
  - Branches
    - Alternatives – branches leaving a square node
    - Chance events – branches leaving a circular node
- Analyze from right to left
  - For each decision, choose the alternative that will yield the greatest return
  - If chance events follow a decision, choose the alternative that has the highest expected monetary value (or lowest expected cost)

# Example – Decision Tree

- A manager must decide on the size of a video arcade to construct. The manager has narrowed the choices to two: large or small. Information has been collected on payoffs, and a decision tree has been constructed. Analyze the decision tree and determine which initial alternative (build small or build large) should be chosen in order to maximize expected monetary value.



# Example – Decision Tree



$$EV_{\text{Small}} = .40(40) + .60(55) = \$49$$

$$EV_{\text{Large}} = .40(50) + .60(70) = \$62$$

**Build the large facility**

# Expected Value of Perfect Information

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- **Expected value of perfect information (EVPI)**
  - The difference between the expected payoff with perfect information and the expected payoff under risk
  - Two methods for calculating EVPI
    - $EVPI = \text{expected payoff under certainty} - \text{expected payoff under risk}$
    - $EVPI = \text{minimum expected regret}$

# Sensitivity Analysis

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- **Sensitivity analysis**
  - Determining the range of probability for which an alternative has the best expected payoff
  - The approach illustrated is useful when there are two states of nature
    - It involves constructing a graph and then using algebra to determine a range of probabilities over which a given solution is best.

# Sensitivity Analysis

Alternative	State of Nature		Slope	Equation
	#1	#2		
A	4	12	$12 - 4 = +8$	$4 + 8P(2)$
B	16	2	$2 - 16 = -14$	$16 - 14P(2)$
C	12	8	$8 - 12 = -4$	$12 - 4P(2)$

